





# The Tale of the ARMA Dynasty

by *Santino del Fava*

“**I**n the mystical land of Johanistan, the plains were inhabited by those known as Time-Series Processes. These strange people were a sequence of observations arranged according to the time of their outcome. There was a division amongst these people: that of stationarity. The non-stationary processes were those whose expected values and autocovariance depended on time. On the other hand, stationary processes were independent of time. For centuries, these two types of processes waged war against each other. But who can forget the siege of Time-Series City twenty years ago?

51

The ARMA dynasty - descendants of a line of stationary processes - held the throne of that once great city. One fateful day, a strange force took root in the city and toppled the ARMA dynasty. No one could identify the process of this strange force, and to this day, it maintains control over the city. Some say the strange force was a non-stationary process. But woe, what's the point of speculation? The reality is that the ARMA dynasty's king, queen, and two sons had been dethroned and killed. Oh, how we meek dwellers of the poor farming village of Autoregressiveville wish for a change in fortunes!”

MA(t) stood and listened to the drunk man's rambling. That was the hundredth time that MA(t) entertained his stories.

"You fancy yourself a historian, don't you, Random Walk?" said MA(t). Random Walk replied, "Show some respect, boy! For the toppling of that great city, yes, I was there. And you, little 20-year-old, what would you know? I was a citizen! Once a stationary man myself, but now look at me, pitiful, non-stationary, and an infinite sum of white noise."

MA(t) was an orphan, found by farmers and raised like one. He had many difficulties growing up and had no time to look beyond his past actions since he found no correlation between his outputs and periods past one lag. One day he stumbled upon Random Walk, drunk in an alley. Drunks were common in their town. Random Walk sent him to buy some booze, and so began their friendship.

"Listen, boy, I'm out of a drink, and there's a shortage in the store. I heard that Old Man Dickey-Fuller's got a special brew. Go atop the hill and snatch me some. We'll share, of course."

52 "Dickey-Fuller... he's a sage. He brews potions, not alcohol. He's a scholar."

"...He's a would-be wizard at best. I need my drink now! But if you don't want to get me some, I can always tell your parents that you've been drinking with me in the dark for years now."

MA(t) agreed to help Random Walk. He reached Old Man Dickey-Fuller's house atop the hill. He snuck in and tried snooping for something he could take back to Random Walk. When he found what he was looking for and headed for the exit, he heard a booming voice call, "MA(t) — the prophecy reads true".

MA(t) turned and saw the old man.

"MA(t), the one who belongs to those stationary greats! MA(t), descendent of the ARMA family."

"How do you know who I am, old man? Leave me be."



“You, boy, are of stationary descent, the one the old one has prophesied about.” The old man continued, “You exist beyond time, and the time is nigh, the time to defeat the force ravaging Time-Series City, the force you are destined to defeat. You see, boy, I have studied that force for years. I have developed a technique to identify his process. My Dickey-Fuller test will determine whether that thing is of stationary or non-stationary origin. If it is non-stationary, we will take first differences to make it stationary”.

53

MA(1) was bewildered. He was a farm boy, not a warrior. But here he was, being told he was a descendant of the legendary ARMA family. The stories Random Walk spoke of weren't mere ramblings as he had initially thought.

“You, boy, are the son of the last ARMA king, sent away as a child. You were orphaned after the city fell to the siege. Now is your time to regain control and bring the stationary ARMA dynasty back to its rightful place amongst the chief authority!”

MA(1) finally spoke, “Old Man Dickey-Fuller, you are known as a wise man, and only for that do I believe you. I'll join you in finding a way to defeat the mysterious process that holds Time-Series City. But first, if I am the King's son, what of my brother?”

“Your brother was known as  $AR(1)$ . He was the king’s eldest son. Days before the siege, he disappeared and has not been heard of since. Once the city is recaptured, we may begin to investigate his fate.”

The two travelled to Time-Series City. The place was shrouded in darkness, and an unsettling air of tension persisted. Once in the city, the old man began his work. He started shouting strange phrases, and  $MA(1)$  watched on as he cast his spells.

“Null hypothesis, you are not rejected! I have considered Case 3 (trend) and seen that your studentised test statistics’  $p$ -value is greater than any reasonable significance level. Mysterious force, you have a unit root; you are non-stationary! Your name is  $ARI(1,1)$ .”

The mysterious force spoke in a sinister tone. His voice was heard, but he was out of view as the city’s darkness hid him.

He called, “Ah! You’re good, old man. But not good enough!”

54 And the mysterious force struck down Old Man Dickey-Fuller.  $MA(1)$  tried to rescue the old man, but it was too late.

The old man quietly said, “Use the differencing technique. That is how you can identify his process”.

$MA(1)$  proceeded to use the first differences on  $ARI(1,1)$ , and with a great struggle, he managed to reveal that the mysterious process was underlined by a stationary  $AR(1)$  process!

This was  $MA(1)$ ’s long-lost brother, but with the corrupting force of non-stationarity removed.

$AR(1)$  finally spoke, “Brother, thank you for saving me from that dark force. The sacking of the city, it was my fault. I was too ambitious and tried to attain power by involving myself with those who were non-stationary. Please forgive me”.

MA(1) forgave AR(1), and soon the two brothers began to rebuild the Time-Series City. As descendants of the ARMA dynasty, they ruled the city together. In honour of Old Man Dickey-Fuller, they taught his hypothesis testing technique to all scholars meant to identify foreign processes. MA(1) returned to his old village of Autoregressiveville as a king, yet humble enough to buy Random Walk one more drink for old-time's sake.

And so, peace was restored in the land of Johanistan, and everyone lived happily ever after.