





Stationaryville and the Two Brothers

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Once upon a time, in a far and magical land called Stationaryville, lived two brothers. The brothers' names were $AR(1)$ and $AR(2)$ and they were part of the prestigious Autoregressive family; one that prided itself on being stationary and invertible in Stationaryville. Their family also liked to be generally linear on things.

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$AR(1)$ was $AR(2)$'s older brother. The brothers loved each other very much and always took care of each other. They did everything together. They played together, laughed together, ate together, and they were best friends.

One day the older brother, $AR(1)$, asked his younger brother if he wanted to go on a Time-Series journey with him — a journey through time. $AR(1)$ asked his younger brother to go on this journey because he realised they were both non-stationary, a condition not allowed in Stationaryville. Because the brothers loved going on adventures together and needed to go on a quest to be stationary, $AR(2)$ decided that he would go on this beautiful Time-Series journey with his brother.

This journey would certainly test their courage and intelligence, so the brothers needed to check a few things about themselves before going on the adventure. The brothers remembered that they had to ensure they were generally linear because they belonged to the Autoregressive family. Furthermore, they realised that they have the power of μ . The power of μ states that they will have an expected value of μ . And no matter what they did, their expected value would be μ .

Brothers raised in the same family are often the same, but more often than not, they are different. These brothers were definitely different in the sense that AR(1) had one lag, and AR(2) had two lags. However, the brothers knew that they could use their differences to make each other stronger, and these differences would help them through the Time-Series journey. The brothers knew that if they worked together, they would be unstoppable.

As the brothers went on the Time-Series journey to obtain stationarity, they stumbled on the land of Invertibility. The land of Invertibility had a wise guardian who guarded its powers. Can you guess what the powers were? It's Invertibility, of course.

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The brothers told the guardian why they were on this journey, and since the guardian was wise and had a kind heart, he decided to grant the brothers the power of Invertibility.

The guardian told them: "Since you belong to the Autoregressive family, and because I see how hard you are both working towards your journey, I will grant both of you the power of Invertibility. But listen to me very carefully. Stick together as brothers and use your new powers wisely."

The brothers thanked the guardian and left the land of Invertibility to continue on their Time-Series journey.

As the brothers progressed on their journey through time, they finally reached the Gate of Stationarity. Guarding this gate and its power was an evil wizard named Station-Nation. Station-Nation was a greedy little wizard who did not like sharing his powers unless his conditions were satisfied.



He was a very picky wizard - a picky and hard-to-please wizard.

Station-Nation heard all about the brothers' problems through the grapevine, and he started taunting them.

"I heard that you boys are non-stationary. How can you not be stationary and live in Stationaryville?" The brothers knew the wizard was right, but they told the wizard they would do whatever it took to become stationary. The wizard saw their eagerness as an opportunity to trap them.

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Station-Nation told them: "Since you two are brothers, but different in your own way, you each have unique powers. You have to use these unique powers to help you gain the power of Stationarity. If you can tell me how to use these powers, I will grant you the gift you seek. But, I know that you will fail."

Even though the brothers were scared, they took the wizard up on his offer, and they thought back to what their parents had told them when they were just young boys: that they were both born with special powers, powers that could make them stationary.

AR(2), thinking on his feet, quickly thought of what could make him stationary. He told the wizard that three things could make him stationary.

AR(2) told the wizard, “If I harness the power of ϕ_1 and ϕ_2 , then I can be stationary. Firstly, I need to ensure that $\phi_2 + \phi_1 < 1$. Secondly, I need to make sure that $\phi_2 + \phi_1 < 1$. Finally, and the cherry on top, is that $-1 < \phi_2 < 1$ ”.

The wizard was surprised that AR(2) passed his test and reluctantly gave him the power of Stationarity.

It was harder for AR(1) to remember his powers, but with the encouragement and support of his younger brother, he remembered: “Aha, you trickster, I know my powers. For me to be stationary, I need to harness my power of ϕ to make sure that $-1 < \phi < 1$.”

The wizard knew that the brother was right, and because he was bound by his powers to grant his promise, he also gave the power of Stationarity to AR(1).

The brothers were overjoyed that they defeated the wizard and managed to get the power of Stationarity to continue living in Stationaryville.

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They returned home and lived happily ever after in Stationaryville.

